A model for sustainable water usage networks design

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Abstract

Both water preparation and treatment systems operation costs can be considerably decreased by optimization of industrial water usage network. The industrial water usage system optimization can be performed by wastewater re-use, by re-allocation of water streams and by recycling without process changes. The aim of the paper is to propose the systematic industrial water usage network retrofit approach based on mathematical programming model. The industrial example of application is included.

Аннотация

Предложена процедура проектирования схем водного хозяйства, обеспечивающая существенное снижение эксплуатационных затрат на очистку природных и сточных вод путем оптимизации системы промышленного водопотребления. Оптимизация системы промышленного водопотребления воды может быть выполнена путем внедрения повторного использования сточных вод, перераспределения потоков воды без внесения изменений в технологию. Разработанная процедура модификации существующих схем водного хозяйства базируется на применении задачи математического программирования. Эффективность предложенного подхода проиллюстрирована промышленным примером.

Keywords: Optimizations, sustainable technology, retrofit design, water re-use, water usage network

1. Introduction

The improvement of environmental and social performance of industry by innovations for optimal resource efficiency and cleaner solutions is key issue to deal with the variety of rising environmental challenges. The transition to more sustainable, resource efficient industrial systems on the basis of eco-innovative products, processes and networks has a lot of the business opportunities along with contributions to effective environmental protection and social progress.

Present increasing standard of living is a challenge for natural environment and natural resources and for water as one of the key resources particularly. The «Water and sanitation» section of Rio+20 outcome document begins with the sentence that water is at the core of sustainable development as it is closely linked to a number of key global challenges. Though water is one of the abundant natural resources, the requirement for water of proper quality has greatly increased through fast development of world industry. Since currently the remarkable strides have been made in treating chemical processes as integrated system, this gives rise to application of process integration methodologies aimed at water consumption reduction in existing process systems.

The problem is most often referred to as water usage network (WUN) design/optimization. In order to make the design process efficient and easy computer software is needed to aid the designer.

Creating software needs a well-formalized systematic design procedure. Similarly to other process system design approaches (Furman and Sahinidis, 2002), there are two broad classes of approaches to design optimal WUN. The water pinch concept (Wang and Smith, 1994), as one of insight-based methods, now is advanced and widely used. However, for large problems with multiple contaminants the application of the pinch method is very difficult and does not ensure even sub-optimal solutions (Alva-Argáez at al., 1999).

The group of systematic methods (Takama at al., 1980; Huang at al., 1999; Koppol at al., 2003; Jeżowski at al., 2006; Lim at al., 2008; Tiana at al., 2008; Xiao Feng at al., 2008; Matijasevic at al., 2010) consists in solving of mathematical programming problem formulated on the basis of WUN superstructure. The main reason of applying systematic optimization-based methods is their robustness.

These methods are more conducive to implementation in software. The main reason of applying systematic optimization-based methods is their robustness, i.e. possibility of calculating the global optimum independent of number of variables (scale of the problem).

Hereafter the robust water usage network design procedure, based on superstructure concept and mathematical programming, is presented and illustrated by an industrial example. The method is focused on retrofit of existing water network.

2. Mathematical model of water usage network

As stated above, our model of water usage system is based on superstructure principle. The scheme of the superstructure is shown in fig .1.

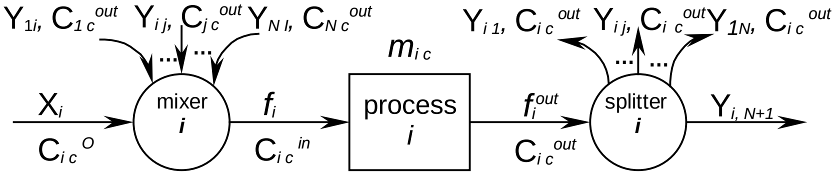

Figure 1 The superstructure for water usage network retrofit design.

The superstructure consists of water using processes, mixers and splitters. All possible WUN structures are embedded in this superstructure. The data commonly required for synthesis scenario are as follows:

1. The set of contaminants, grouped by analogous properties.
2. For each water using process:
   - maximum feasible input concentration for each contaminant;
   - maximum feasible output concentration for each contaminant;
   - mass load for each contaminant;
   - water losses and gains.
3. For each external freshwater source:
   - upper limit on water source supply capability;

- the quality of water (concentrations of contaminants);
- the specific cost of water.

4. For entire process – the nominal cost of wastewater reuse from one process to another (pipeline transportation costs and so on), cost of wastewater treatment.

Several authors, e.g. Dunn at al. (2001), insisted that for retrofit case contaminants input and output concentrations and flow-rates through processes should be kept at the same values as in existing processes. In this paper we follow these recommendations since we consider them well suited for industrial requirements. Hence, we require that for each water using process input concentration of contaminants and flow-rate of water stream should be given. Notice, that since mass loads of contaminant are given, the outlet concentrations from operations are fixed and known for retrofit case.

To present the optimization model of the superstructure some necessary definitions and symbols are presented in the following.

Let No be the number of water usage operations; Ns be the number of freshwater sources; Nc be the number of contaminants.

Let X_{ij} [kg/h] be the flow-rate of water stream from source #*i* to operation #*j*; Y_{ij} [kg/h] is the flow-rate of stream from output of operation #*i* to input of operation #*j* (see the superstructure in fig. 1)

The superstructure model of WUN consists of the system of equations, which are mainly mass balances of units in the superstructure. Notice that the model presented in the following is general and does not make use of data specific for retrofit case.

The model of freshwater source

$$\sum_{i=1}^{No} X_{ij} \le F^{s}_{\max_j}, \quad j=\overline{1,Ns}, \tag{1}$$

where $F^{s}_{\max_j}$ is water supply ability of freshwater source #*j*.

The model of mixer (the input point to process)

a) overall balance

$$\sum_{s=1}^{Ns} X_{si} + \sum_{j=1}^{No} Y_{ij} = f_i, \quad i=\overline{1,No} \tag{2}$$

b) mass balances of each contaminant

$$\begin{aligned} &\sum_{l=1}^{Ns}\left(C^{0}_{lc}\,X_{li}\right)+\sum_{j=1}^{No}\left(C^{out}_{jc}\,Y_{ij}\right)= \\ &\qquad = C^{in}_{ic}\left(\sum_{l=1}^{Ns}X_{li}+\sum_{j=1}^{No}Y_{ij}\right), \\ &i=\overline{1,No}, \quad c=\overline{1,Nc} \end{aligned} \tag{3}$$

where C^{0}_{lc} – the concentration of contaminant c in water of freshwater source #*l*.

The model of water using process

a) overall mass balance

$$f_i^{OUT} = f_i - f_i^{LOSS} + f_i^{GAINS}, \qquad i = \overline{1, No} \tag{4}$$

b) balances for each contaminant (in general case)

$$C_{ic}^{out} = \varphi_{ijc}(C_{ic}^{in}, \Delta C_{ic}, f_i, m_{ic}), \quad i = \overline{1, No}; \quad c = \overline{1, Nc}$$

where φ_{ijc} is a certain, given function; m_{ic} is a mass load for contaminant $\# c$; ΔC_{ic} is a value of concentration change in process #*i* (for contaminant $\# c$)

$$C_{ic}^{out} = \varphi_{ijc}(C_{ic}^{in}, \Delta C_{ic}, f_i, m_{ic}), \quad i = \overline{1, No}; \quad c = \overline{1, Nc}$$

For a simple case

$$C_{ic}^{out} = C_{ic}^{in} + \Delta C_{ic}, \qquad i = \overline{1, No}; \quad c = \overline{1, Nc} \tag{5}$$

$$\Delta C_{ic} = \frac{f_i}{m_{ic}}, \qquad i = \overline{1, No}; \quad c = \overline{1, Nc} \tag{6}$$

The model of splitters (the output point to process): overall mass balance

$$\sum_{j=1}^{No} Y_{ij} + Y_0 = f_i^{OUT}, \qquad i = \overline{1, No} \tag{7}$$

The mass balances for the sequence: mixer-process-splitter:

a) streams

$$\sum_{i=1}^{No}\sum_{j=1}^{Ns} X_{ij} + \left(\sum_{i=1}^{No}\sum_{j=1}^{No} Y_{ij}\right) = \\ = \left(\sum_{i=1}^{No}\sum_{j=1}^{No} Y_{ij}\right) + \sum_{i=1}^{No} Y_{i0} + \left(\sum_{i=1}^{No} f_i^{GAINS} - \sum_{i=1}^{No} f_i^{LOSS}\right) \tag{8}$$

b) for each contaminant ($c = \overline{1, Nc}$):

$$\sum_{l=1}^{Ns} C_{lc}^{0} + \sum_{i=1}^{No} \Delta C_{ic} = \sum_{i=1}^{No} C_{ic}^{out} \tag{9}$$

Technological constraints ($i = \overline{1, No}; \quad c = \overline{1, Nc}$):

$$C_{ic}^{in} \le C_{icMAX}^{in} \tag{10}$$

$$C_{ic}^{out} \le C_{icMAX}^{out} \tag{11}$$

In order to ensure that the global minimum usage of fresh water will be met one can add the additional constraint:

$$\sum_{i=1}^{n}\sum_{l=1}^{k} X_{li} \le f_{MIN}^{PINCH} \tag{12}$$

Parameter f_{MIN}^{PINCH} denotes the minimum usage of freshwater. The value of this parameter can be calculated beforehand by some methods available in the literature, e.g. water pinch approach from Wang and Smith (1994).

3. Optimization criterion

The most common performance index for WUN applied in the literature is cost of fresh water consumed in the network (notice that in case of a single fresh water source the goal function is simply the usage of fresh water):

$$\min \qquad Z = \sum_{l=1}^{k} \left(W_l \sum_{i=1}^{n} X_{li} \right) \tag{13}$$

where W_l is unit cost of freshwater from source $\#l$.

It is well known that the goal function influences the performance of optimization for nonlinear problems in particular. Hence, we performed some numerical experiments with various other goal functions. It is clear that in order to reduce fresh water cost a network should reuse as much water as possible. Hence, we applied the goal function (13) in order to minimize fresh water consumption and to maximize water reuse:

$$\min \qquad Z = \sum_{l=1}^{Ns} \left(W_l \sum_{i=1}^{No} X_{li} \right) - \left(\sum_{i=1}^{No} \sum_{j=1}^{No} Y_{ji} \right) \tag{14}$$

We have also developed other goal functions similar to (14). They are listed below.

$$\min \qquad Z = \sum_{l=1}^{k} \left(W_l \sum_{i=1}^{n} X_{li} \right) - \left(\sum_{i=1}^{No} \sum_{j=1}^{No} Y_{ji} \right) - \left(\sum_{i=1}^{n} \frac{\sum_{j=1}^{n} Y_{ji}}{\sum_{l=1}^{k} W_l X_{li}} \right) \tag{15}$$

$$\min \qquad Z = \sum_{l=1}^{k} \left(W_l \sum_{i=1}^{n} X_{li} \right) - \left(\sum_{i=1}^{n} \frac{\sum_{j=1}^{n} Y_{ji}}{\sum_{l=1}^{k} W_l X_{li}} \right) \tag{16}$$

$$\min \qquad Z = -\left(\sum_{i=1}^{n} \frac{\sum_{j=1}^{n} Y_{ji}}{\sum_{l=1}^{k} W_l X_{li}} \right) \tag{17}$$

All the performance indices (13)-(17) were tested for the problems from the literature. The goal function (14) suggested by us ensures the highest performance of the optimization (though the performance of goal function (13) is of similar order) and this goal function was applied in further investigations.

In order to account for cost of piping the additional term has to be added to fresh water cost. To calculate this term one needs unit costs of pipes. Since such parameters are difficult to estimate precisely in industry we propose the use of relative cost of pipes. Parameters c_{ij} in matrix C can be simply taken as distances between processes *i* and *j*.

Relative reusing costs factors v_{ij} can be calculated as follows:

$$\overline{v_{ij}} = \frac{c_{ij}}{\max\limits_{i,j}\left\{c_{ij}\right\}} \tag{18}$$

One can notice that since $\overline{v_{ij}}$ are relative cost coefficients, hence cost parameters for freshwater W_{ij} in (13) should also be treated as relative unit cost of fresh water. Such relative costs are defined by:

$$\varpi_l = \frac{W_l}{\sum\limits_{s=1}^{k} W_s} \tag{19}$$

In order to account for cost of connections for water reuse in modified goal functions (13)–(17) we suggest to multiply parameters Y_{ij} by reciprocals of coefficients $\overline{v_{ij}}$:

$$v_{ij} = 1 \Big/ \overline{v_{ij}}, \qquad i, j = \overline{1, No} \tag{20}$$

Thus, the modified goal function (14) becomes:

$$\min \qquad Z = \sum_{l=1}^{Ns}\left(\varpi_l \sum_{i=1}^{No} X_{li}\right) - \left(\sum_{i=1}^{No}\sum_{j=1}^{No} v_{ij} Y_{ji}\right) \tag{21}$$

4. Case study

Let’s illustrate the proposed methodology by the case study from pulp-and-paper industry. Technological input data are presented in table 1.

Table 1 Water usage units (limiting data)

#	**Process**	**Mass load kg/h**	**C_{in}, mg O_2/ dm^3**	**C_{out}, mg O_2/ dm^3**	**Freshwater flowrate, m^3/h**
1	Pulping	24800.0	500	5000	5511.1
2	Bleach wash	50.0	100	400	166.7
3	Papermaking machine (I)	-18225.0	5000	500	4050.0
4	Papermaking machine (II)	-6480.0	5000	200	1350.0
5	Fiber washing	3.3	20	100	41.6
6	Blanket washing	21.7	100	500	54.2
7	Drum washing	37.5	300	600	125.0
8	Mercerization	10.0	100	220	83.3
9	Degumming	4.8	200	2500	2.1
10	Desizing	35.4	300	3700	10.4
	Total				11394.4

Goal function:

$$\min \quad Z = x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{110} - Y_{11} - Y_{12} - Y_{13} -$$
$$Y_{14} - Y_{15} - Y_{16} - Y_{17} - Y_{18} - Y_{19} - Y_{110} - Y_{21} - Y_{22} - Y_{23} - Y_{24} - Y_{25} - Y_{26} - Y_{27} - Y_{28} -$$
$$Y_{29} - Y_{210} - Y_{31} - Y_{32} - Y_{33} - Y_{34} - Y_{35} - Y_{36} - Y_{37} - Y_{38} - Y_{39} - Y_{310} - Y_{41} - Y_{42} -$$
$$Y_{43} - Y_{44} - Y_{45} - Y_{46} - Y_{47} - Y_{48} - Y_{49} - Y_{410} - Y_{51} - Y_{52} - Y_{53} - Y_{54} - Y_{55} - Y_{56} -$$
$$Y_{57} - Y_{58} - Y_{59} - Y_{510} - Y_{61} - Y_{62} - Y_{63} - Y_{64} - Y_{65} - Y_{66} - Y_{67} - Y_{68} - Y_{69} - Y_{610} -$$
$$Y_{71} - Y_{72} - Y_{73} - Y_{74} - Y_{75} - Y_{76} - Y_{77} - Y_{78} - Y_{79} - Y_{710} - Y_{81} - Y_{82} - Y_{83} - Y_{84} -$$
$$Y_{85} - Y_{86} - Y_{87} - Y_{88} - Y_{89} - Y_{810} - Y_{91} - Y_{92} - Y_{93} - Y_{94} - Y_{95} - Y_{96} - Y_{97} - Y_{98} -$$
$$Y_{99} - Y_{910} - Y_{101} - Y_{102} - Y_{103} - Y_{104} - Y_{105} - Y_{106} - Y_{107} - Y_{108} - Y_{109} - Y_{1010}$$

Constraints:

$$\begin{cases}
\dfrac{5000 \cdot Y_{11} + 500 \cdot Y_{21} + 200 \cdot Y_{31} + 500 \cdot Y_{41} + 600 \cdot Y_{51} + 100 \cdot Y_{61}}{x_{11} + Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} + Y_{61} + Y_{71} + Y_{81} + Y_{91} + Y_{101}} + \\
\dfrac{220 \cdot Y_{71} + 400 \cdot Y_{81} + 2500 \cdot Y_{91} + 3700 \cdot Y_{101}}{x_{11} + Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} + Y_{61} + Y_{71} + Y_{81} + Y_{91} + Y_{101}} \le 500 \\
\dfrac{5000 \cdot Y_{12} + 500 \cdot Y_{22} + 200 \cdot Y_{32} + 500 \cdot Y_{42} + 600 \cdot Y_{52} + 100 \cdot Y_{62}}{x_{12} + Y_{12} + Y_{22} + Y_{32} + Y_{42} + Y_{52} + Y_{62} + Y_{72} + Y_{82} + Y_{92} + Y_{102}} + \\
\dfrac{220 \cdot Y_{72} + 400 \cdot Y_{82} + 2500 \cdot Y_{92} + 3700 \cdot Y_{102}}{x_{12} + Y_{12} + Y_{22} + Y_{32} + Y_{42} + Y_{52} + Y_{62} + Y_{72} + Y_{82} + Y_{92} + Y_{102}} \le 5000 \\
\dfrac{5000 \cdot Y_{13} + 500 \cdot Y_{23} + 200 \cdot Y_{33} + 500 \cdot Y_{43} + 600 \cdot Y_{53} + 100 \cdot Y_{63}}{x_{13} + Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} + Y_{63} + Y_{73} + Y_{83} + Y_{93} + Y_{103}} + \\
\dfrac{220 \cdot Y_{73} + 400 \cdot Y_{83} + 2500 \cdot Y_{93} + 3700 \cdot Y_{103}}{x_{13} + Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} + Y_{63} + Y_{73} + Y_{83} + Y_{93} + Y_{103}} \le 5000 \\
\dfrac{5000 \cdot Y_{14} + 500 \cdot Y_{24} + 200 \cdot Y_{34} + 500 \cdot Y_{44} + 600 \cdot Y_{54} + 100 \cdot Y_{64}}{x_{14} + Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} + Y_{64} + Y_{74} + Y_{84} + Y_{94} + Y_{104}} + \\
\dfrac{220 \cdot Y_{74} + 400 \cdot Y_{84} + 2500 \cdot Y_{94} + 3700 \cdot Y_{104}}{x_{14} + Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} + Y_{64} + Y_{74} + Y_{84} + Y_{94} + Y_{104}} \le 100 \\
\dfrac{5000 \cdot Y_{15} + 500 \cdot Y_{25} + 200 \cdot Y_{35} + 500 \cdot Y_{45} + 600 \cdot Y_{55} + 100 \cdot Y_{65}}{x_{15} + Y_{15} + Y_{25} + Y_{35} + Y_{45} + Y_{55} + Y_{65} + Y_{75} + Y_{85} + Y_{95} + Y_{105}} + \\
\dfrac{220 \cdot Y_{75} + 400 \cdot Y_{85} + 2500 \cdot Y_{95} + 3700 \cdot Y_{105}}{x_{15} + Y_{15} + Y_{25} + Y_{35} + Y_{45} + Y_{55} + Y_{65} + Y_{75} + Y_{85} + Y_{95} + Y_{105}} \le 300 \\
\dfrac{5000 \cdot Y_{16} + 500 \cdot Y_{26} + 200 \cdot Y_{36} + 500 \cdot Y_{46} + 600 \cdot Y_{56} + 100 \cdot Y_{66}}{x_{16} + Y_{16} + Y_{26} + Y_{36} + Y_{46} + Y_{56} + Y_{66} + Y_{76} + Y_{86} + Y_{96} + Y_{106}} + \\
\dfrac{220 \cdot Y_{76} + 400 \cdot Y_{86} + 2500 \cdot Y_{96} + 3700 \cdot Y_{106}}{x_{16} + Y_{16} + Y_{26} + Y_{36} + Y_{46} + Y_{56} + Y_{66} + Y_{76} + Y_{86} + Y_{96} + Y_{106}} \le 20 \\
\dfrac{5000 \cdot Y_{17} + 500 \cdot Y_{27} + 200 \cdot Y_{37} + 500 \cdot Y_{47} + 600 \cdot Y_{57} + 100 \cdot Y_{67}}{x_{17} + Y_{17} + Y_{27} + Y_{37} + Y_{47} + Y_{57} + Y_{67} + Y_{77} + Y_{87} + Y_{97} + Y_{107}} + \\
\dfrac{220 \cdot Y_{77} + 400 \cdot Y_{87} + 2500 \cdot Y_{97} + 3700 \cdot Y_{107}}{x_{17} + Y_{17} + Y_{27} + Y_{37} + Y_{47} + Y_{57} + Y_{67} + Y_{77} + Y_{87} + Y_{97} + Y_{107}} \le 100 \\
\dfrac{5000 \cdot Y_{18} + 500 \cdot Y_{28} + 200 \cdot Y_{38} + 500 \cdot Y_{48} + 600 \cdot Y_{58} + 100 \cdot Y_{68}}{x_{18} + Y_{18} + Y_{28} + Y_{38} + Y_{48} + Y_{58} + Y_{68} + Y_{78} + Y_{88} + Y_{98} + Y_{108}} + \\
\dfrac{220 \cdot Y_{78} + 400 \cdot Y_{88} + 2500 \cdot Y_{98} + 3700 \cdot Y_{108}}{x_{18} + Y_{18} + Y_{28} + Y_{38} + Y_{48} + Y_{58} + Y_{68} + Y_{78} + Y_{88} + Y_{98} + Y_{108}} \le 100 \\
\dfrac{5000 \cdot Y_{19} + 500 \cdot Y_{29} + 200 \cdot Y_{39} + 500 \cdot Y_{49} + 600 \cdot Y_{59} + 100 \cdot Y_{69}}{x_{19} + Y_{19} + Y_{29} + Y_{39} + Y_{49} + Y_{59} + Y_{69} + Y_{79} + Y_{89} + Y_{99} + Y_{109}} + \\
\dfrac{220 \cdot Y_{79} + 400 \cdot Y_{89} + 2500 \cdot Y_{99} + 3700 \cdot Y_{109}}{x_{19} + Y_{19} + Y_{29} + Y_{39} + Y_{49} + Y_{59} + Y_{69} + Y_{79} + Y_{89} + Y_{99} + Y_{109}} \le 200 \\
\dfrac{5000 \cdot Y_{110} + 500 \cdot Y_{210} + 200 \cdot Y_{310} + 500 \cdot Y_{410} + 600 \cdot Y_{510} + 100 \cdot Y_{610}}{x_{110} + Y_{110} + Y_{210} + Y_{310} + Y_{410} + Y_{510} + Y_{610} + Y_{710} + Y_{810} + Y_{910} + Y_{1010}} + \\
\dfrac{220 \cdot Y_{710} + 400 \cdot Y_{810} + 2500 \cdot Y_{910} + 3700 \cdot Y_{1010}}{x_{110} + Y_{110} + Y_{210} + Y_{310} + Y_{410} + Y_{510} + Y_{610} + Y_{710} + Y_{810} + Y_{910} + Y_{1010}} \le 3
\end{cases}$$

$$\begin{cases}
Y_{11}+Y_{12}+Y_{13}+Y_{14}+Y_{15}+Y_{16}+Y_{17}+Y_{18}+Y_{19}+Y_{110}+Y_{10}\le 5555.1\\
Y_{21}+Y_{22}+Y_{23}+Y_{24}+Y_{25}+Y_{26}+Y_{27}+Y_{28}+Y_{29}+Y_{210}+Y_{20}\le 4050\\
Y_{31}+Y_{32}+Y_{33}+Y_{34}+Y_{35}+Y_{36}+Y_{37}+Y_{38}+Y_{39}+Y_{310}+Y_{30}\le 1350\\
Y_{41}+Y_{42}+Y_{43}+Y_{44}+Y_{45}+Y_{46}+Y_{47}+Y_{48}+Y_{49}+Y_{410}+Y_{40}\le 54.2\\
Y_{51}+Y_{52}+Y_{53}+Y_{54}+Y_{55}+Y_{56}+Y_{57}+Y_{58}+Y_{59}+Y_{510}+Y_{50}\le 125\\
Y_{61}+Y_{62}+Y_{63}+Y_{64}+Y_{65}+Y_{66}+Y_{67}+Y_{68}+Y_{69}+Y_{610}+Y_{60}\le 41.6\\
Y_{71}+Y_{72}+Y_{73}+Y_{74}+Y_{75}+Y_{76}+Y_{77}+Y_{78}+Y_{79}+Y_{710}+Y_{70}\le 83.3\\
Y_{81}+Y_{82}+Y_{83}+Y_{84}+Y_{85}+Y_{86}+Y_{87}+Y_{88}+Y_{89}+Y_{810}+Y_{80}\le 166.7\\
Y_{91}+Y_{92}+Y_{93}+Y_{94}+Y_{95}+Y_{96}+Y_{97}+Y_{98}+Y_{99}+Y_{910}+Y_{90}\le 2.1\\
Y_{101}+Y_{102}+Y_{103}+Y_{104}+Y_{105}+Y_{106}+Y_{107}+Y_{108}+Y_{109}+Y_{1010}+Y_{100}\le 10.42
\end{cases}$$

$$\begin{cases}
x_{11}+Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51}+Y_{61}+Y_{71}+Y_{81}+Y_{91}+Y_{101}=5511.1\\
x_{12}+Y_{12}+Y_{22}+Y_{32}+Y_{42}+Y_{52}+Y_{62}+Y_{72}+Y_{82}+Y_{92}+Y_{102}=4050\\
x_{13}+Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53}+Y_{63}+Y_{73}+Y_{83}+Y_{93}+Y_{103}=1350\\
x_{14}+Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54}+Y_{64}+Y_{74}+Y_{84}+Y_{94}+Y_{104}=54.2\\
x_{15}+Y_{15}+Y_{25}+Y_{35}+Y_{45}+Y_{55}+Y_{65}+Y_{75}+Y_{85}+Y_{95}+Y_{105}=125\\
x_{16}+Y_{16}+Y_{26}+Y_{36}+Y_{46}+Y_{56}+Y_{66}+Y_{76}+Y_{86}+Y_{96}+Y_{106}=41.6\\
x_{17}+Y_{17}+Y_{27}+Y_{37}+Y_{47}+Y_{57}+Y_{67}+Y_{77}+Y_{87}+Y_{97}+Y_{107}=83.3\\
x_{18}+Y_{18}+Y_{28}+Y_{38}+Y_{48}+Y_{58}+Y_{68}+Y_{78}+Y_{88}+Y_{98}+Y_{108}=166.7\\
x_{19}+Y_{19}+Y_{29}+Y_{39}+Y_{49}+Y_{59}+Y_{69}+Y_{79}+Y_{89}+Y_{99}+Y_{109}=2.1\\
x_{110}+Y_{110}+Y_{210}+Y_{310}+Y_{410}+Y_{510}+Y_{610}+Y_{710}+Y_{810}+Y_{910}+Y_{1010}=10.4
\end{cases}$$

$$\forall i,j,l:\quad X_{l\,i}>0$$

$$Y_{i\,j}\ge 0\quad i=\overline{1,10},\quad j=\overline{1,10},\ l=1$$

The possible structure of WUN corresponding to the results of the model optimization is presented in fig. 2.

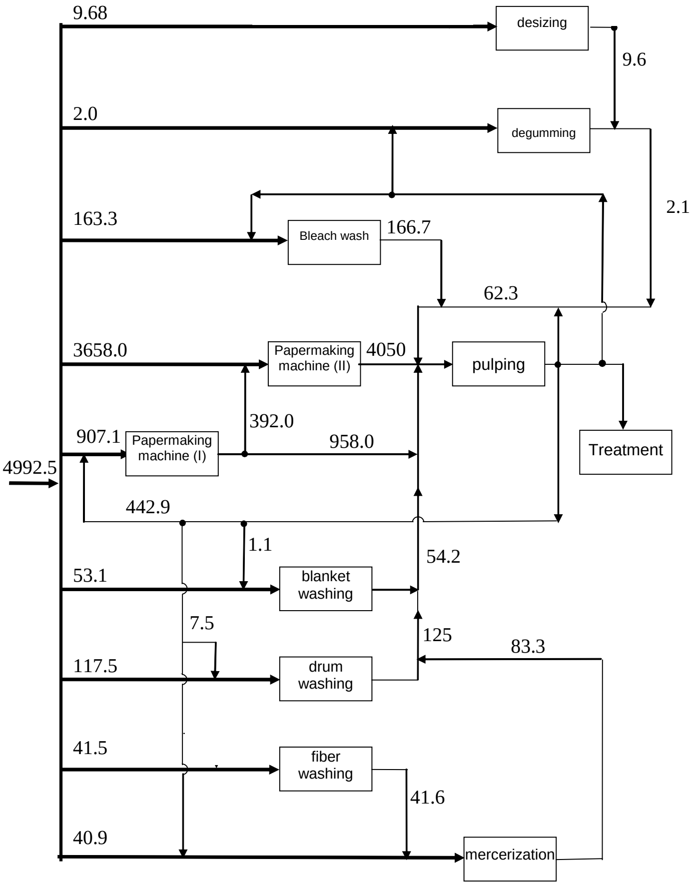

Figure 2 Possible structure of WUN of pulp-and-paper plant.

In the optimized network the amount of wastewater to treatment has decreased by 56.18 %.

5. Conclusions

The methodology of industrial water networks retrofit design was developed. The stage of optimal network design by a MPP solution was considered in more details. Mathematical programming model with provision for costs of fresh and reused water, possibilities of wastewater reuse was developed. Industrial illustrative case study presented in the paper proved the efficiency of the approach.

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